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Real time radiative corrections to charged particle decay laws

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Abstract. The real time exponential decay laws for meta-stable charged particles are shown to require radiative corrections. The methods employed are well known to be valid for radiatively correcting Breit-Wigner line shapes. Radiative corrections contribute substantially to precision life time measurements of muons and pions when initially stopped in condensed matter.

1 Introduction

It is well known[1] that the exponential decay law[2] for the survival probability $P_0(t)$ of a meta-stable particle,

$$P_0(t) \approx \exp(-\Gamma t), \qquad (1)$$

arises from an energy distribution[3] which is nearly Lorentzian in shape

$$dW_0(E) \approx \Big(\frac{\hbar\Gamma}{2\pi}\Big)\Big(\frac{dE}{(E-E_0)^2+(\hbar\Gamma/2)^2}\Big)\ ; \qquad (2)$$

i.e.

$$P_0(t) = \Big|\int e^{-iEt/\hbar} dW_0(E)\Big|^2 \quad \text{(exponential decay)}. \qquad (3)$$

It is also very well known that when charged particles are involved in a decay process, the Lorentzian energy distribution $dW_0(E)$ of the meta-stable particle must be radiatively corrected[4][5] [6] to a new energy distribution $dW(E)$. The new distribution has a much more "skewed" line shape than that of a simple Lorentzian. It then follows that the survival probability also has a radiative correction in real time; i.e.

$$P(t) = \Big|\int e^{-iEt/\hbar} dW(E)\Big|^2 \quad \text{(radiatively corrected)}. \qquad (4)$$

Our purpose is to discuss the real time consequences of radiation in the decay of meta-stable charged particles.

Previous to this work, the notion that radiative corrections have important implications[7] for observations of quantum noise in α-decays and β-decays in heavy nuclei was theoretically developed[8] for real time counting rates. Strong experimental evidence for quantum noise in nuclear β-decay counting rates has been reported[9][10].

The time scales of quantum noise observations in nuclear physics are of the order of a few hours to a few days. These long times are still much less than the nuclear life times, which in turn are of the order of a few years. For the application of radiative corrections to real time decay laws in high energy physics, e.g. for the weak decay

$$\pi^+ \to \mu^+ + \nu_\mu, \qquad (5)$$

or the weak decay

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu, \qquad (6)$$

the available absolute measurement time scales are considerably reduced. Thus, the methods of detecting real time radiative corrections should be somewhat different from those methods used for nuclear weak decays.

In Sect. 2, the radiatively corrected survival probability $P(t)$ will be calculated from the mean number of soft photons $dN(\omega)$ radiated into the bandwidth $d\omega$. The properties of the resulting distribution are discussed in Sect. 3, in terms of the time dependent transition rate for decay $\gamma(t)$, defined as

$$P(t) = \exp\Big(-\int_0^t \gamma(s)ds\Big). \qquad (7)$$

In Sect. 3 we illustrate the computation of the time dependent transition rate for a simple two body decay of a charged particle, e.g. (5). In Sect. 4 we discuss the soft photon emission coupling strength as a function of photon frequency and the velocity of the produced charged particle both for insulators and conductors. We discuss the soft photon emission in both the Bremsstrahlung and Cerenkov regimes. In Sect. 5, we find (for times *short* on the scale of inverse soft photon frequencies) that there exists a "hot spot" in the decay rate; i.e. the decay rate per unit time $\gamma(t)$ has a sharp peak for short times. In the long time limit, the decay rate settles down to Γ, which determines the intrinsic life time. In the vacuum, as well as in materials, the decay rates exhibit a long time inverse

t asymptotic time dependence which may be described by

$$\gamma(t) = \Gamma + \Big(\frac{2\beta_0}{t}\Big) + \ ... \ \text{ as } t \to \infty. \tag{8}$$

In (8), the over all coupling strength β_0 for the soft photons may be computed from the mean number of radiated photons $dN(\omega)$ in a bandwidth $d\omega$; It is

$$\beta_0 = \lim_{\omega\to 0}\ \omega\Big(\frac{dN(\omega)}{d\omega}\Big). \tag{9}$$

In the concluding Sect. 6, we discuss why the notion of radiative corrections to real time decay measurements in high energy physics is surely worthy of further experimental study.

2 Energy distributions and survival probabilities

Let Ψ denote the internal wave function of an unstable charged particle in the center of mass frame. The energy distribution of the state is given by

$$dW(E) = \big(\Psi, \delta(E - H)\Psi\big)dE. \tag{10}$$

The survival amplitude for the state Ψ is given by

$$\mathcal{S}(t) = \big(\Psi, e^{-iHt/\hbar}\Psi\big), \tag{11}$$

which is rigorously related to the energy distribution via

$$\mathcal{S}(t) = \int e^{-iEt/\hbar} dW(E). \tag{12}$$

The survival probability

$$P(t) = |\mathcal{S}(t)|^2 \tag{13}$$

is thus given by (4). If, in a two body decay of a charged particle, $d\mathcal{P}(\omega)$ is the probability of emitting soft photon radiation in the energy interval $\hbar d\omega$, then the radiatively corrected renormalization $dW_0(E) \to dW(E)$ is computed via the energy convolution

$$\Big(\frac{dW(E)}{dE}\Big) = \int \Big(\frac{dW_0(E - \hbar\omega)}{dE}\Big) d\mathcal{P}(\omega). \tag{14}$$

If, during the decay, there are a mean number $\bar{n}_k$ of photons radiated into mode k with Poisson statistics, then

$$\frac{d\mathcal{P}(\omega)}{d\omega} = \sum_{\{n\}} \Big\{ \prod_k \Big(\frac{\bar{n}_k^{n_k} e^{-\bar{n}_k}}{n_k!}\Big)\Big\} \delta\Big(\omega - \sum_k n_k \omega_k\Big). \tag{15}$$

Employing the generating function in the time domain

$$\int_0^\infty e^{-i\omega t} d\mathcal{P}(\omega) = e^{-\chi(t)}, \tag{16}$$

implies the simplification

$$\chi(t) = \int_0^\infty \big(1 - e^{-i\omega t}\big) dN(\omega), \tag{17}$$

where $dN(\omega)$ is the mean number of photons radiated into a bandwidth $d\omega$; i.e. (15) and (16) imply (17) with

$$dN(\omega) = \Big(\sum_k \bar{n}_k \delta(\omega - \omega_k)\Big) d\omega. \tag{18}$$

Furthermore, (1),(3),(4),(14) and (16) imply

$$P(t) = \exp\big(-\Gamma t - 2\ \Re e\ \chi(t)\big). \tag{19}$$

(17) and (19) for the radiatively corrected survival probability are the central results of this section. Equations (7), (17) and (19) imply that the radiatively corrected transition rate per unit time $\gamma(t)$ as a function of time is related to the mean number of soft photons $dN(\omega)$ radiated into a bandwidth $d\omega$ via

$$\gamma(t) = \Gamma + 2\int_0^\infty \omega \sin(\omega t) dN(\omega). \tag{20}$$

The asymptotic (8) follows from (9) and (20).

3 Computation of transition rates

In order to compute $\big(dN(\omega)/d\omega\big)$, and thereby $\gamma(t)$, for the decay of a charged particle stopped in matter, we employ Schwinger's photon propagator method[11]. For a charged particle moving on a path C, the self action due to virtual photons is given by

$$S = \frac{e^2}{2c}\int_C\int_C \mathcal{D}_{\mu\nu}(x - y) dx^\mu dy^\nu, \tag{21}$$

where $\mathcal{D}_{\mu\nu}(x - y)$ is the photon propagator in the condensed matter[12] wherein the original charged particle was stopped. Employing the k-space representation

$$\mathcal{D}_{\mu\nu}(x - y) = \int D_{\mu\nu}(k) e^{ik\cdot(x-y)} \Big(\frac{d^4k}{(2\pi)^4}\Big). \tag{22}$$

with

$$L^\mu(k) = \int_C e^{ik\cdot x} dx^\mu, \tag{23}$$

the action

$$S = \frac{e^2}{2c}\int L^\mu(k) D_{\mu\nu}(k) L^\nu(k)^* \Big(\frac{d^4k}{(2\pi)^4}\Big). \tag{24}$$

One may compute the mean number of radiated photons using

$$N = 2\Im m\big(S/\hbar\big). \tag{25}$$

With the fine structure constant

$$\alpha = \Big(\frac{e^2}{\hbar c}\Big), \tag{26}$$

we have

$$N = \alpha \int \Im m\Big(L^\mu(k) D_{\mu\nu}(k) L^\nu(k)^*\Big)\Big(\frac{d^4k}{(2\pi)^4}\Big). \tag{27}$$

If v_i represents the initial four velocity of a particle (before a two body decay) and v_f represents the final recoil four velocity of the produced charged particle after decay, then one easily obtains the usual expression[4] for $L(k)$; i.e.

$$L(k) = i\Big\{\Big(\frac{v_f}{k \cdot v_f}\Big) - \Big(\frac{v_i}{k \cdot v_i}\Big)\Big\}. \tag{28}$$

For the *vacuum* radiation distribution case

$$D^{vac}_{\mu\nu}(k) = \Big(\frac{4\pi\eta_{\mu\nu}}{k^2 - i0^+}\Big), \tag{29}$$

so that (37), (38) and (39) imply the well known[4] *vacuum* radiated photon distribution

$$d^3N_{vac}(\mathbf{k}) = \Big(\frac{\alpha d^3\mathbf{k}}{4\pi^2|\mathbf{k}|}\Big)\Big\{\Big(\frac{v_f}{k \cdot v_f}\Big) - \Big(\frac{v_i}{k \cdot v_i}\Big)\Big\}^2. \tag{30}$$

In the vacuum rest frame of the charged particle (before decay), we find the usual result, as in (18),

$$dN(\omega) = \Big(\int \delta(\omega - c|\mathbf{k}|)d^3N_{vac}(\mathbf{k})\Big)\, d\omega = \beta\Big(\frac{d\omega}{\omega}\Big). \tag{31}$$

For example, if in the rest frame of the charged particle (before decay), a single final state charge has velocity $\mathbf{v}$, then

$$\beta(\mathbf{v}) = \Big(\frac{\alpha}{\pi}\Big)\Big\{\Big(\frac{c}{|\mathbf{v}|}\Big)\ln\Big(\frac{c+|\mathbf{v}|}{c-|\mathbf{v}|}\Big) - 2\Big\}. \tag{32}$$

Equations (20), (31) and (32) yield

$$\gamma(t) = \Gamma + \Big(\frac{2\beta(\mathbf{v})}{t}\Big), \quad \text{(vacuum decay)}. \tag{33}$$

The radiative corrections for a charged particle decay, when the particle has been stopped in condensed matter, are somewhat more subtle.

4 Decay of particles stopped in condensed matter

Condensed matter systems are often described (for complex frequency ζ in the upper half $\Im m(\zeta) > 0$ plane) by a dielectric response function $\varepsilon(\zeta)$, or by a conductivity response function $\sigma(\zeta)$. These are related by

$$\epsilon(\zeta) = 1 + \Big(\frac{4\pi i\sigma(\zeta)}{\zeta}\Big). \tag{34}$$

The above description may be incorporated into the temporal gauge photon propagator, $\zeta \to (|\omega| + i0^+)$, written[12] as

$$\mathbf{D}(\mathbf{k},\omega) = \Big(\frac{4\pi}{|\mathbf{k}|^2 - (\omega/c)^2\varepsilon(|\omega|+i0^+)}\Big)\Big\{\mathbf{1} - \Big(\frac{c^2\mathbf{kk}}{\omega^2\varepsilon(|\omega|+i0^+)}\Big)\Big\}. \tag{35}$$

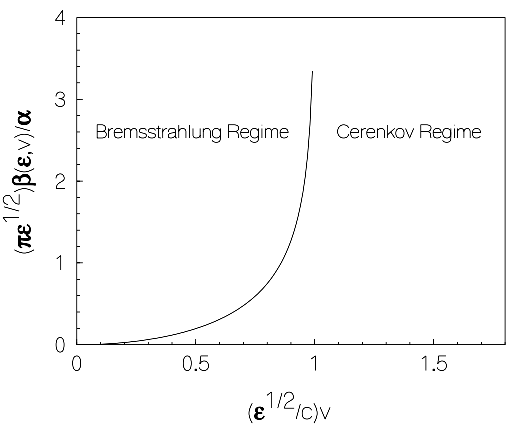

Fig. 1. Bremsstrahlung coupling strength as a function of velocity for an insulator from (38) and (42)

Employing the temporal gauge (35) in the evaluation of the number of radiated photons in (27) and (28) yields

$$\beta(\omega,\mathbf{v}) = \omega\Big(\frac{dN(\omega,\mathbf{v})}{d\omega}\Big), \tag{36}$$

i.e.

$$\beta(\omega,\mathbf{v}) = \Big(\frac{\omega\alpha}{\pi c}\Big)\Im m\int\Big(\frac{\mathbf{v}\cdot\mathbf{D}(\mathbf{k},\omega+i0^+)\cdot\mathbf{v}}{(\mathbf{k}\cdot\mathbf{v}-\omega)^2}\Big)\Big(\frac{d^3\mathbf{k}}{(2\pi)^3}\Big). \tag{37}$$

The condensed matter version of the vacuum (32) is found (after some tedious integration) to be

$$\beta(\omega,\mathbf{v}) = \Re e\Big\{\Big(\frac{\alpha}{\pi\sqrt{\varepsilon(\omega+i0^+)}}\Big)\mathcal{F}\big(z(\omega+i0^+), z^*(\omega+i0^+)\big)\Big\} \tag{38}$$

where

$$z(\zeta) = \Big(\frac{c}{v\sqrt{\varepsilon(\zeta)}}\Big), \tag{39}$$

$$\mathcal{F}(z,z^*) = \Big(\frac{\mathcal{G}(z)-\mathcal{G}(z^*)}{z-z^*}\Big) \tag{40}$$

and

$$\mathcal{G}(z) = \Big(\frac{z^2-1}{2}\Big)\ln\Big(\frac{z+1}{z-1}\Big) - z. \tag{41}$$

In the limit of real values for z

$$\lim_{y\to 0}\mathcal{F}(x+iy) = \Big\{x\ln\Big(\frac{x+1}{x-1}\Big) - 2\Big\}, \quad \text{if } |x| > 1. \tag{42}$$

In a (somewhat unphysical) model where the dielectric response ε is independent of frequency, there will be a Bremsstrahlung regime for velocities obeying as shown in the above Fig. 1;

$$|\mathbf{v}| < \Big(\frac{c}{\sqrt{\varepsilon}}\Big) \quad \text{Brehmsstrahlung}. \tag{43}$$

The value of the coupling strength in the insulating material Bremsstrahlung regime is given by

$$\beta(\mathbf{v},\varepsilon) = \Big(\frac{\alpha}{\pi\varepsilon}\Big)\Big\{\Big(\frac{c}{|\mathbf{v}|}\Big)\ln\Big(\frac{c+\sqrt{\varepsilon}|\mathbf{v}|}{c-\sqrt{\varepsilon}|\mathbf{v}|}\Big) - 2\Big\}. \tag{44}$$

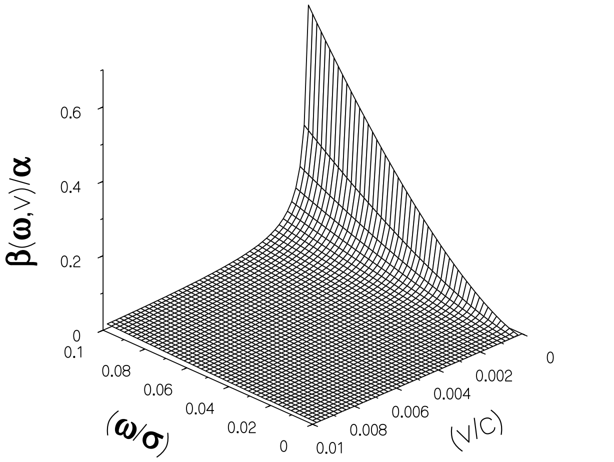

Fig. 2. Coupling strength for a non-relativistic particle in a conductor

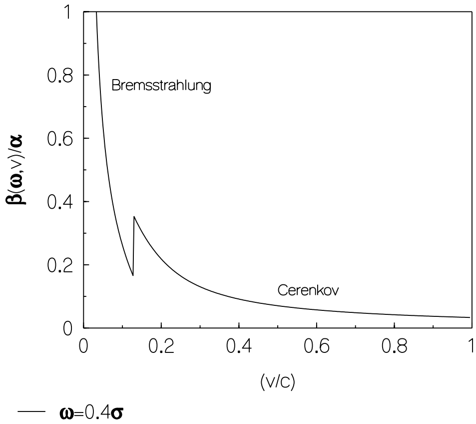

Fig. 3. The almost discontinuous transition from the Bremsstrahlung to the Cerenkov regimes in a conductor

In the high velocity regime there will be Cerenkov radiation.

$$c > |\mathbf{v}| > \Big(\frac{c}{\sqrt{\varepsilon}}\Big) \quad \text{Cerenkov}. \tag{45}$$

However, the Cerenkov radiation regime can be discussed carefully only in models where the full complex dielectric response is taken into account. The dissipation in physical continuous media implies a finite $\beta(\omega, \mathbf{v})$ in all regimes.

For example, in a model for an Ohm's law conducting material, the dielectric response function obeys

$$\varepsilon(\omega) = 1 + \Big(\frac{4\pi i\sigma}{\omega}\Big) + \dots . \tag{46}$$

For a non-relativistic particle the resulting coupling strength $\beta(\omega, \mathbf{v})$ is plotted in Fig. 2.

One may look at high velocity (for some values of (ω/σ)) to note the almost discontinuous jump from the Bremsstrahlung regime to the Cerenkov regime in an Ohm's law metal. This is shown for $(\omega/\sigma) = 0.4$ in Fig. 3. For a high conductivity metal, $\sigma \sim 10^{18}/sec$. Thus, $\hbar\sigma \sim 1\ KeV$ which establishes the order of magnitude of the maximum frequency with which to define "soft photons" in this Ohm's law conducting model.

Finally, for a conductor in the low frequency limit

$$\Big(\frac{1}{\sqrt{\varepsilon}}\Big) \to (1-i)\sqrt{\Big(\frac{\omega}{8\pi\sigma}\Big)}, \quad \text{as} \ \ \omega \to 0, \tag{47}$$

and

$$z \to (1-i)\Big(\frac{c}{v}\Big)\sqrt{\Big(\frac{\omega}{8\pi\sigma}\Big)}, \quad \text{as} \ \ \omega \to 0. \tag{48}$$

. From (38), (40), (41), (47) and (48) it follows that

$$\beta_0(\mathbf{v}) = \lim_{\omega\to 0} \beta(\omega, \mathbf{v}) = \Big(\frac{\alpha|\mathbf{v}|}{2c}\Big), \quad \text{(conductor)}, \tag{49}$$

independent of the conductivity σ.

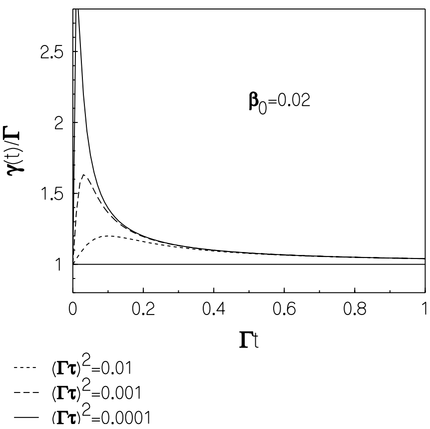

Fig. 4. Transition rate "hot spots" for various cut-offs

5 Hot spots and long time tails

A typical model for the mean number of photons $dN(\omega)$ in a bandwidth $d\omega$ employs an exponential cut-off for high frequency; it reads

$$dN(\omega) = \beta_0 e^{-\omega\tau}\Big(\frac{d\omega}{\omega}\Big). \tag{50}$$

where $(1/\tau)$ is the frequency cut-off. From (20) and (50) it follows that

$$\gamma(t) = \varGamma + 2\beta_0\Big(\frac{t}{t^2+\tau^2}\Big) \tag{51}$$

may be used to compute the time dependent transition rate in (7). If $t >> \tau$, then the time dependence of $\gamma(t)$ is given by (8). Ultimately $\lim_{t\to\infty}\gamma(t) = \varGamma$, i.e. the intrinsic decay rate. For very short times the intrinsic rate also dominates; i.e. $\lim_{t\to 0}\gamma(t) = \varGamma$. However, for intermediate times there exists a peak or "hot spot" in which the decay rate may increase substantially. The situation is shown in Fig. 4 which should hold true for both the vacuum decay and for decays in an insulator. For the insulating case β_0 is plotted in Fig. 1.

By plotting $\gamma(t)$ in (7), one emphasizes the short time deviations from the uniform in time transition rate $\varGamma$.

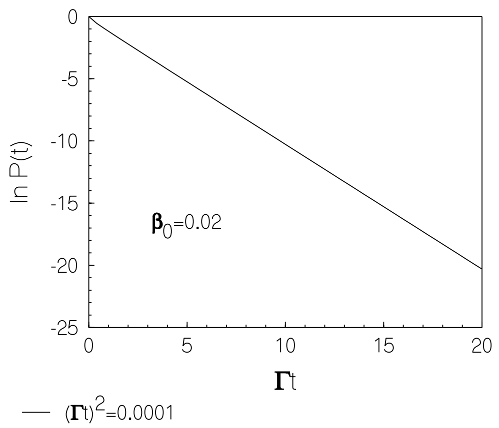

Fig. 5. In a direct plot of ln P(t), the hot spot is not very prominent

More conventionally[13][14], experimentalists directly plot *ln* $P(t)$. In such plots, the short time hot spot appears not merely less pronounced, but in reality *hardly visible*. Thus, it may have escaped some deserved attention.

From (7) and (51), it is found in simple Bremsstrahlung models that

$$P(t) = \Big(\frac{\tau^2}{t^2+\tau^2}\Big)^{\beta_0} exp\big(-\Gamma t\big) \tag{52}$$

To see what is involved, we plot the survival probability $P(t)$ in the above Fig. 5. The short time hot spot for $(\Gamma\tau)^2 = 0.0001$, which is so very obvious when plotted as in Fig. 4, is not at all so obvious when plotted as in Fig. 5. Both curves are mathematically equivalent in accordance with (7). To explore the short time hot spot on an experimental level, one must examine the data in some detail during the period of the first life time of the survival probability, (say) at the early times $0 < t < (0.5/\Gamma)$. For precision life time measurements, one tries a much wider time interval, (say) $0 < t < (10/\Gamma)$. Such wide time intervals may mask important material effects which may have an effect on experimental precision. According to (52), the exponential decay law $exp(-\Gamma t)$ has a materials dependent prefactor $\big(1+(t/\tau)^2\big)^{-\beta_0}$ when explored over some twenty life times.

Equation (52) holds true for both conductors and insulators when the produced charged particle is in the Bremsstrahlung regime. The Cerenkov regime for insulators is a bit more subtle. If one may define[15] for an insulator, a Debye relaxation time τ_D, via

$$\Im m\big(\varepsilon(\omega+i0^+)\big) \to \varepsilon(\omega\tau_D), \quad \text{as} \quad \omega \to 0, \tag{53}$$

then in the Cerenkov regime $v > (c/\sqrt{\varepsilon})$, and in the low frequency limit $\omega \to 0$,

$$\beta_C(\omega, |\mathbf{v}|) \to \Big(\frac{\alpha v}{c}\Big)\Big(\frac{1}{\omega\tau_D}\Big)\Big\{1-\Big(\frac{c^2}{\varepsilon v^2}\Big)\Big\}. \tag{54}$$

From (20) and (54) it follows that in the insulating Cerenkov regime, the observed transition rate is renormalized to

$$\gamma_C = \Gamma + \Big(\frac{\pi\alpha v}{c\tau_D}\Big)\Big\{1-\Big(\frac{c^2}{\varepsilon v^2}\Big)\Big\}. \tag{55}$$

Finally, the case of an insulator with a fractal low frequency behavior[16] having exponent η,

$$\Im m\big(\varepsilon(\omega+i0^+)\big) \to \varepsilon\Big(\frac{\omega\tau_D}{|\omega\tau_D|^\eta}\Big), \quad \text{as} \quad \omega \to 0, \tag{56}$$

leads to

$$\gamma_C(t,\eta) = \Gamma + \varphi(\eta)\Big(\frac{\alpha v}{c\tau_D}\Big)\Big\{1-\Big(\frac{c^2}{\varepsilon v^2}\Big)\Big\}\Big(\frac{\tau_D}{t}\Big)^\eta, \tag{57}$$

where

$$\varphi(\eta) = 2\Gamma(\eta)\sin\Big(\frac{\pi\eta}{2}\Big), \quad \Gamma(\eta) = \int_0^\infty x^\eta e^{-x}\Big(\frac{dx}{x}\Big). \tag{58}$$

From (7) and (57) follows the Cerenkov fractal exponent survival probability

$$P_C(t) = \exp\Big(-\Gamma t - \Phi_C(t,\eta,\tau_D)\Big), \tag{59}$$

where

$$\Phi_C(t,\eta,\tau_D) = \Big(\frac{\varphi(\eta)\alpha v}{(1-\eta)c}\Big)\Big\{1-\Big(\frac{c^2}{\varepsilon v^2}\Big)\Big\}\Big(\frac{t}{\tau_D}\Big)^{(1-\eta)}, \tag{60}$$

and which exhibits (for short times) a stretched exponential form. Note as $\eta \to 0$, (60) becomes equivalent to (55).

6 Concluding remarks

The dielectric suppression of Bremsstrahlung in materials is a well known experimental effect[17]. In this work, deviations from the exponential laws in real time triggered by soft photons have been studied for decaying charged particles in materials. Under the *standard assumption* of factorizability of the "dynamical" energy distribution from the soft photon emission spectrum as given in (14), we have derived the transition rates for decays in insulating and conducting materials, both in the Bremsstrahlung and Cerenkov regimes. Several interesting results emerge some of which are listed below.

If a particle decays in a conducting material, a novel discontinuity is predicted to occur. We show in Fig. 3, that as the velocity of the charged particle produced in the decay increases from low values (bremsstrahlung region), the radiative coupling strength (and hence the transition rate) first decreases rapidly, shows a sharp discontinuity as it enters the Cerenkov region and then continues to decrease more slowly. Much care would be needed to experimentally observe such a discontinuity since it is rather sharp and hence confined to a very narrow velocity range of the produced particle.

Another prediction concerns "hot spots" and long time tails occurring both for the vacuum as well as an insulator. As exhibited in Fig. 4, the transition rate has a well defined maximum (hot spot) for intermediate times and is substantially different from its asymptotic value. On the other hand, the same effect is shown in Fig. 5 to be completely washed out in a standard logarithmic plot of the survival probability data commonly presented by experimentalists. Thus, evidence, if any, for such an effect must be sought out through a careful study of the transition rate in the short time interval, say $0 < t < (0.5/\Gamma)$.

The Cerenkov regime for insulators is also of particular interest since it leads to decays of the "stretched exponential" form. The fractal exponent in the absorptive part of the low frequency limit of the dielectric constant is shown to be directly related to the radiative exponent in the real time decay [see (56-60)]. It would be worthwhile to check it experimentally.

In view of the above predicted deviations from purely exponential decays, we urge that a concentrated, systematic and precise experimental study of the transition rate be undertaken, both in the vacuum as well as in diverse materials for different decay particle speeds. Exponential decays and Poisson statistics are almost axiomatic in experimental particle physics. Thus, any deviations are surely of fundamental interest.

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